Lect5

Linear wire antennas

Dr. Gehan Sami

INFINITESIMAL DIPOLE $\ell \leq \lambda/50$

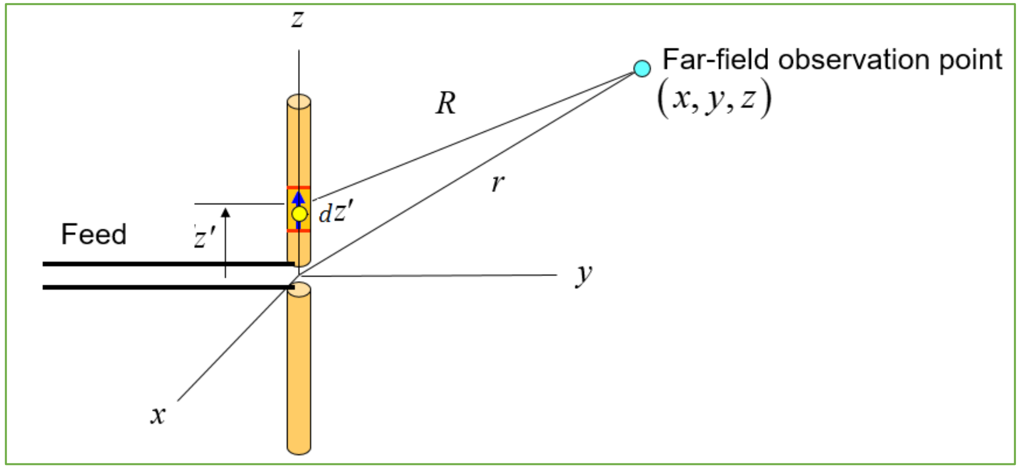
$$\mathbf{I}(z') = \hat{\mathbf{a}}_z I_0$$

$$\mathbf{A}(x, y, z) = \frac{\mu}{4\pi} \int_C \mathbf{I}_e(x', y', z') \frac{e^{-jkR}}{R} \, dl'$$

$$R = \sqrt{(x - x')^2 + (y - y')^2 + (z - z')^2} = \sqrt{x^2 + y^2 + z^2}$$

$$= r = \text{constant}$$

$$dl' = dz'$$



$$\mathbf{A}(x, y, z) = \hat{\mathbf{a}}_z \frac{\mu I_0}{4\pi r} e^{-jkr} \int_{-l/2}^{+l/2} dz' = \hat{\mathbf{a}}_z \frac{\mu I_0 l}{4\pi r} e^{-jkr}$$

$$A_\theta = -A_z \sin\theta = -\frac{\mu I_0 l e^{-jkr}}{4\pi r} \sin\theta$$

$$\mathbf{E} = -j\omega \mathbf{A}$$

$$E_\theta = j\eta \frac{k I_0 l \sin\theta}{4\pi r} e^{-jkr}$$

RADIATION RESISTANCE

$$W_r = \frac{\eta}{8}\left|\frac{I_0 l}{\lambda}\right|^2 \frac{\sin^2\theta}{r^2}$$

$$P = \int_0^{2\pi}\int_0^{\pi} W_r r^2 \sin\theta \, d\theta \, d\phi = \eta\frac{\pi}{3}\left|\frac{I_0 l}{\lambda}\right|^2$$

$$P_{\text{rad}} = \frac{1}{2}|I_0|^2 R_r$$

$$R_r = \eta\left(\frac{2\pi}{3}\right)\left(\frac{l}{\lambda}\right)^2 = 80\pi^2\left(\frac{l}{\lambda}\right)^2$$

λ

Example 4.1

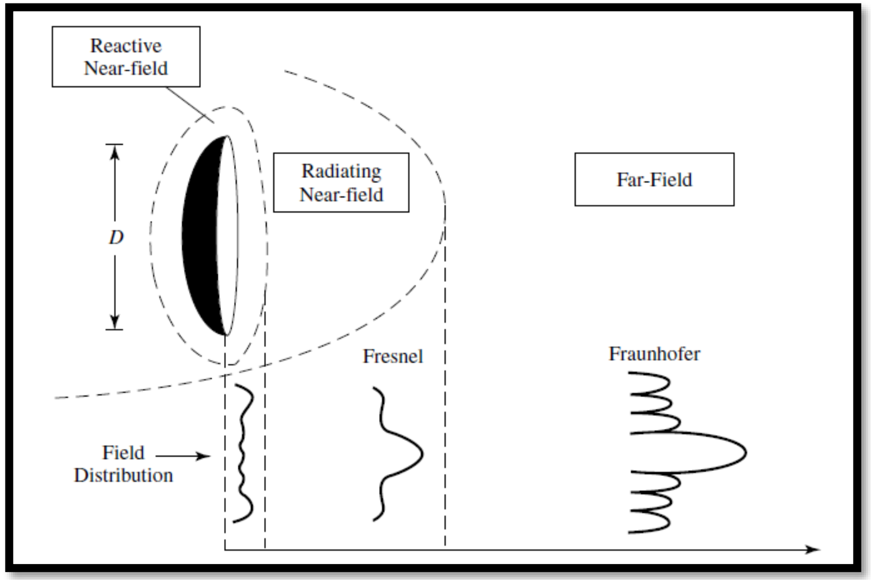
Find the radiation resistance of an infinitesimal dipole whose overall length is $l = \lambda/50$.

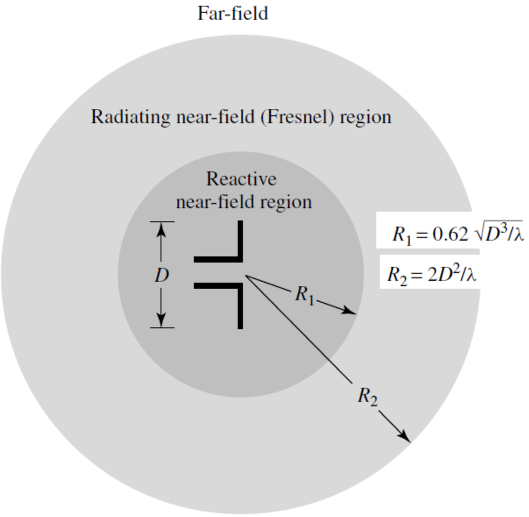
Solution: Using (4-19)

$$R_r = 80\pi^2\left(\frac{l}{\lambda}\right)^2 = 80\pi^2\left(\frac{1}{50}\right)^2 = 0.316 \text{ ohms}$$

Since the radiation resistance of an infinitesimal dipole is about 0.3 ohms, it will present a very large mismatch when connected to practical transmission lines, many of which have characteristic impedances of 50 or 75 ohms. The reflection efficiency (e_r) and hence the overall efficiency (e_0) will be very small.

FIELD REGIONS OF AN ANTENNA





Field regions of an antenna.

- **Far field zone**:

1-field components are transverse to radial direction from antenna, and all power flow is directed radially outward.

2-shape of radiation pattern is independent on distance.

- **Near field zone**:

1-field components may not transverse to radial direction from antenna and power is not entirely radial.

2-shape of radiation pattern is dependent on distance.

$$H_r = H_\theta = 0.$$

$$H_\phi = j\frac{kI_0 l\sin\theta}{4\pi r}\left[1 + \frac{1}{jkr}\right]e^{-jkr}.$$

Given in the Exam

$$E_r = \eta\frac{I_0 l\cos\theta}{2\pi r^2}\left[1 + \frac{1}{jkr}\right]e^{-jkr}$$

$$E_\theta = j\eta\frac{kI_0 l\sin\theta}{4\pi r}\left[1 + \frac{1}{jkr} - \frac{1}{(kr)^2}\right]e^{-jkr}$$

$$E_\phi = 0$$

Near-Field ($kr \ll 1$) Region

$$E_r \simeq -j\eta\frac{I_0 l e^{-jkr}}{2\pi kr^3}\cos\theta \qquad E_\theta \simeq -j\eta\frac{I_0 l e^{-jkr}}{4\pi kr^3}\sin\theta$$

$$E_\phi = H_r = H_\theta = 0 \qquad H_\phi \simeq \frac{I_0 l e^{-jkr}}{4\pi r^2}\sin\theta$$

Intermediate-Field ($kr > 1$) Region

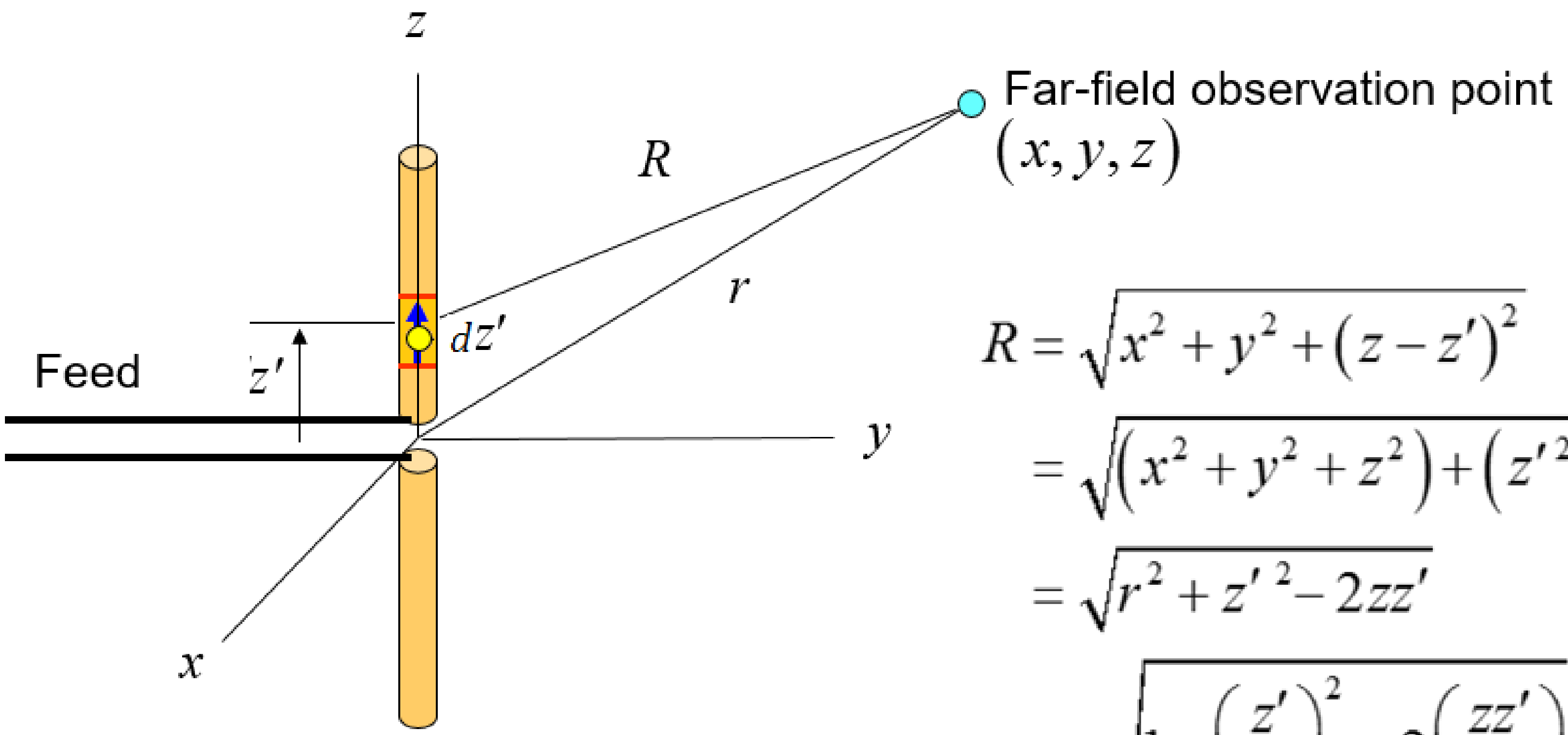
$$E_r \simeq \eta\frac{I_0 l e^{-jkr}}{2\pi r^2}\cos\theta \qquad E_\theta \simeq j\eta\frac{kI_0 l e^{-jkr}}{4\pi r}\sin\theta$$

$$E_\phi = H_r = H_\theta = 0 \qquad H_\phi \simeq j\frac{kI_0 l e^{-jkr}}{4\pi r}\sin\theta$$

Far-Field ($kr \gg 1$) Region

$$E_\theta \simeq j\eta\frac{kI_0 l e^{-jkr}}{4\pi r}\sin\theta \qquad H_\phi \simeq j\frac{kI_0 l e^{-jkr}}{4\pi r}\sin\theta$$

$$E_r \simeq E_\phi = H_r = H_\theta = 0$$



$$R = \sqrt{x^2 + y^2 + (z - z')^2}$$

$$= \sqrt{(x^2 + y^2 + z^2) + (z'^2) - 2(zz')}$$

$$= \sqrt{r^2 + z'^2 - 2zz'}$$

$$= r\sqrt{1 + \left(\frac{z'}{r}\right)^2 - 2\left(\frac{zz'}{r^2}\right)}$$

$$= r\sqrt{1 + \left(\frac{z'}{r}\right)^2 - 2\left(\frac{r\cos\theta\, z'}{r^2}\right)}$$

$$R = r\left[1+\underbrace{\left(\frac{-2rz'\cos\theta + z'^2}{r^2}\right)}_{x}\right]^{1/2} = r[1+x]^{1/2}$$

$$R = r\left[1+\frac{1}{2}x-\frac{1}{8}x^2+\frac{1}{16}x^3\ldots\ldots\right]$$

$$R = r - z'\cos\theta + \frac{1}{r}\left(\frac{z'^2}{2}\sin^2\theta\right) + \frac{1}{r^2}\left(\frac{z'^3}{2}\cos\theta\sin^2\theta\right)$$

Given in the Exam

Max at $\theta=90^{o}$

Max at $\theta=54.74^{o}$ $\cos\theta\sin^2\theta=0.385$

To find reactive near field region

$$\left.\frac{kz'^3}{2r^2}\cos\theta\sin^2\theta\right|_{\substack{z'=l/2\\ \theta=\tan^{-1}\sqrt{2}}} = \frac{\pi}{\lambda}\frac{l^3}{8r^2}\,0.385 \leq \frac{\pi}{8}$$

$$r \geq 0.62\sqrt{l^3/\lambda}$$

DIRECTIVITY

$$U = r^2 W_{av} = \frac{\eta}{8}\left(\frac{I_0 l}{\lambda}\right)^2 \sin^2\theta \qquad P_{rad} = \eta\frac{\pi}{3}\left|\frac{I_0 l}{\lambda}\right|^2$$

$$D = 4\pi\frac{U}{P_{rad}} = \frac{3}{2}\sin^2\theta$$

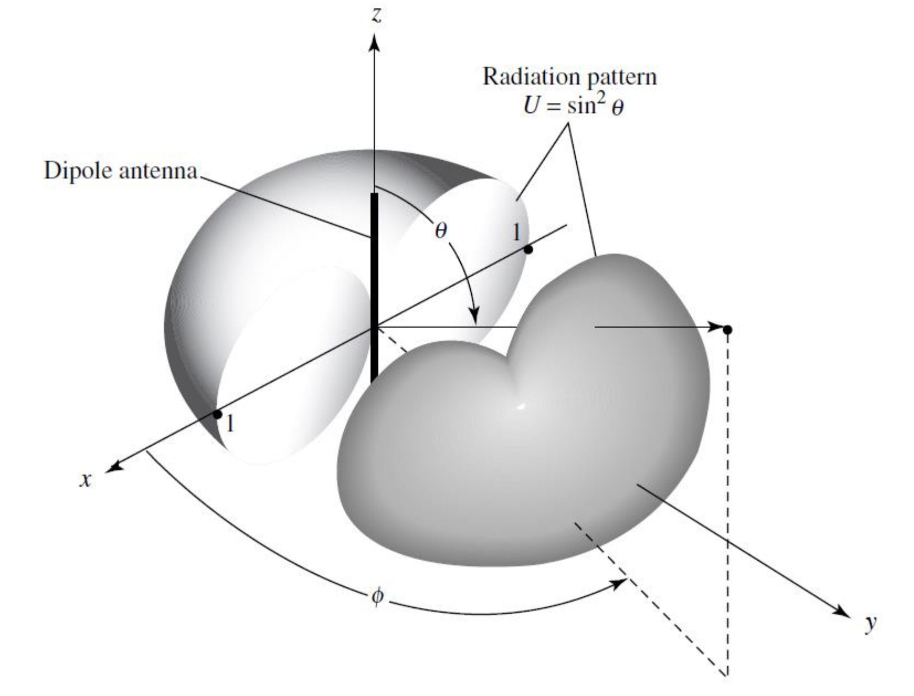


Figure 4.3 Three-dimensional radiation pattern of infinitesimal dipole.

OR

$$D_o = \frac{4\pi}{\int_0^{2\pi}\int_0^{\pi} \sin^2\theta \sin\theta \, d\theta \, d\Phi} = \frac{4\pi}{2\pi\left(\frac{4}{3}\right)} = \frac{3}{2}$$

EFFECTIVE APERTURE

$$A_{em} = \left(\frac{\lambda^2}{4\pi}\right) D_0 = \frac{3\lambda^2}{8\pi}$$

HALF POWER BEAMWIDTH

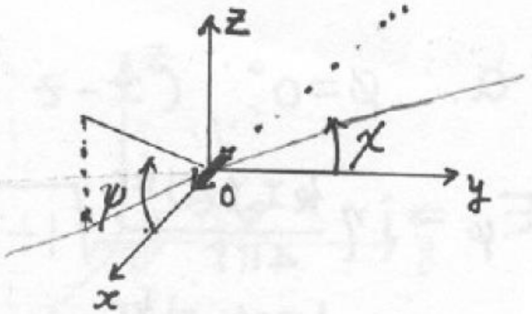
HPBW=90^o

INFINITESIMAL DIPOLE ALONG X AXIS

4-1. a.

$$\sin\psi = \sqrt{1-\cos^2\psi} = \sqrt{1-|\hat{a}_x\cdot\hat{a}_r|^2}$$

$$= \sqrt{1-(\sin\theta\cdot\cos\phi)^2}$$



In far-zone fields

$$E_\psi \simeq j\eta\frac{kI_0\cdot l e^{-jkr}}{4\pi r}\cdot\sin\psi = j\eta\frac{kI_0 l e^{-jkr}}{4\pi r}\sqrt{1-(\sin\theta\cdot\cos\phi)^2}$$

b. $U = U_0(1-\sin^2\theta\cos^2\phi)$

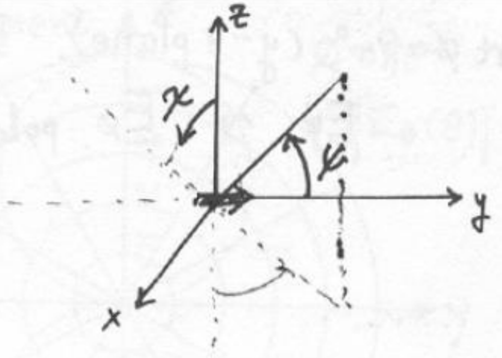
$$\therefore P_{rad} = U_0\int_0^{2\pi}\int_0^{\pi}(1-\sin^2\theta\cdot\cos^2\phi)\cdot\sin\theta\, d\theta\, d\phi$$

$$= U_0\int_0^{2\pi}\int_0^{\pi}\sin\theta\, d\theta\, d\phi - \int_0^{\pi}(\sin^2\theta)\cdot\sin\theta\, d\theta\int_0^{2\pi}\cos^2\phi\, d\phi = U_0\cdot\frac{8\pi}{3}$$

$$D_0 = \frac{4\pi\cdot U_0}{U_0\cdot\frac{8\pi}{3}} = \frac{3}{2} = 1.5$$

INFINITESIMAL DIPOLE ALONG Y AXIS

4-2.



a. $\sin\psi = \sqrt{1-\cos^2\psi} = \sqrt{1-|\hat{a}_y\cdot\hat{a}_r|^2}$

$= \sqrt{1-\sin^2\theta\cdot\sin^2\phi}$

In far-zone fields

$$E_\psi \simeq j\eta\frac{kI_0\ell e^{-jkr}}{4\pi r}\cdot\sin\psi = j\eta\frac{kI_0\ell e^{-jkr}}{4\pi r}\cdot\sqrt{1-\sin^2\theta\cdot\sin^2\phi}$$

$$H_\chi \simeq \frac{E_\psi}{\eta} \simeq j\frac{kI_0 e^{-jkr}\cdot\ell}{4\pi r}\sqrt{1-\sin^2\theta\sin^2\phi}$$

b. $U = U_0(1-\sin^2\theta\sin^2\phi)$

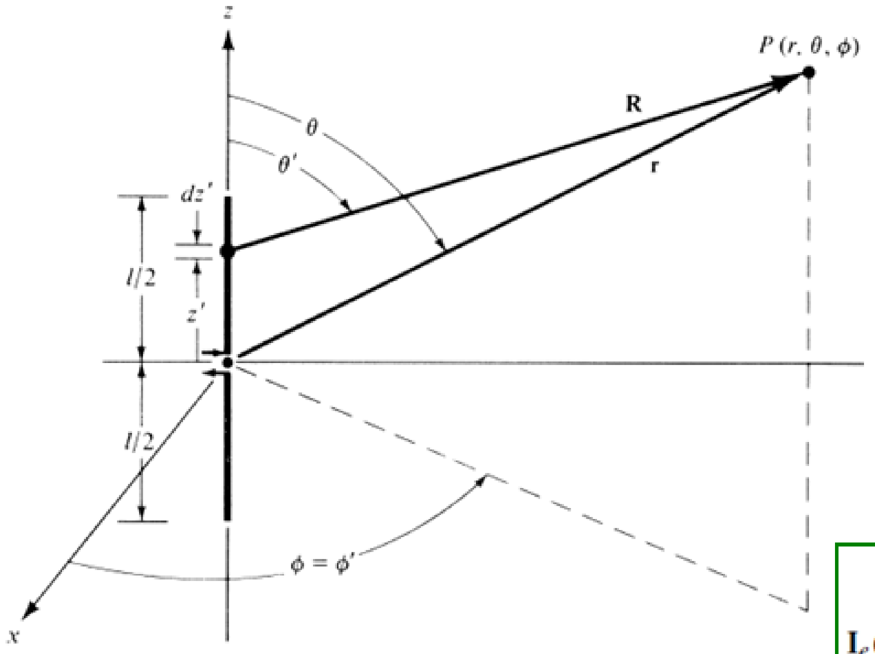
$$P_{rad} = U_0\int_0^{2\pi}\int_0^{\pi}(1-\sin^2\theta\cdot\sin^2\phi)\sin\theta\, d\theta\, d\phi = U_0\int_0^{2\pi}\left[\int_0^{\pi}\sin\theta - \sin^3\theta\cdot\sin^2\phi\, d\theta\right]d\phi$$

$$= U_0\left[\int_0^{2\pi}2\,d\phi - \frac{4}{3}\int_0^{2\pi}\sin^2\phi\, d\phi\right] = U_0\left[4\pi - \frac{4}{3}\pi\right] = \frac{8}{3}\pi\cdot U_0$$

$$D_0 = \frac{4\pi\cdot U_0}{U_0\cdot\frac{8\pi}{3}} = \frac{3}{2} = 1.5$$

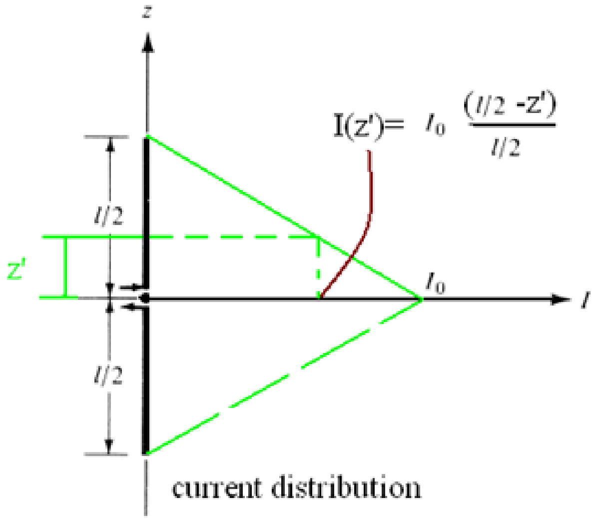
Small dipole Antenna

- Small dipole $\lambda/50 < \ell < \lambda/10$



R≈r in magnitude and phase (max error for phase=18^0 at l=λ/10)

Dipole and geometry



R=r
Max phase error
<180/8=22.5°

$$\mathbf{I}_e(x', y', z') = \begin{cases} \hat{\mathbf{a}}_z I_0 \left(1 - \frac{2}{l} z'\right), & 0 \le z' \le l/2 \\ \hat{\mathbf{a}}_z I_0 \left(1 + \frac{2}{l} z'\right), & -l/2 \le z' \le 0 \end{cases}$$

where I_0 = constant.

$$\mathbf{A}(x, y, z) = \frac{\mu}{4\pi} \int_C \mathbf{I}_e(x', y', z') \frac{e^{-jkR}}{R} dl'$$

$$\mathbf{A}(x, y, z) = \frac{\mu}{4\pi} \left[\hat{\mathbf{a}}_z \int_{-l/2}^{0} I_0 \left(1 + \frac{2}{l} z'\right) \frac{e^{-jkR}}{R} dz' + \hat{\mathbf{a}}_z \int_{0}^{l/2} I_0 \left(1 - \frac{2}{l} z'\right) \frac{e^{-jkR}}{R} dz' \right]$$

$$\mathbf{A} = \hat{\mathbf{a}}_z \frac{1}{2} \left[\frac{\mu I_0 l e^{-jkr}}{4\pi r} \right]$$

which is half obtained by infinitesimal dipole
so Each of Far fields are half of infinitesimal
u is 1/4*of infinitesimal,prad is 1/4 of infinitesimal
D same as infinitesimal as it= 4π.u/p_{rad}

Wire Antenna (cont.)

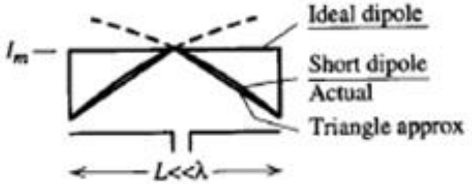
characteristics of some dipole antennas

Dipole Type	Length	Current	Pattern	HP	D	D (dB)	R_r (Ω)
Ideal	$L \ll \lambda$	Uniform	$\sin\theta$	90°	1.5	1.76	$80\pi^2\left(\frac{L}{\lambda}\right)^2$
Short	$L \ll \lambda$	Triangle	$\sin\theta$	90°	1.5	1.76	$20\pi^2\left(\frac{L}{\lambda}\right)^2$
Half-wave	$L = 0.5\,\lambda$	Sinusoid	$\dfrac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta}$	78°	1.64	2.15	~70

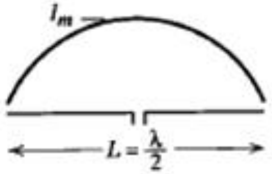
Current Distribution

Ideal

Short

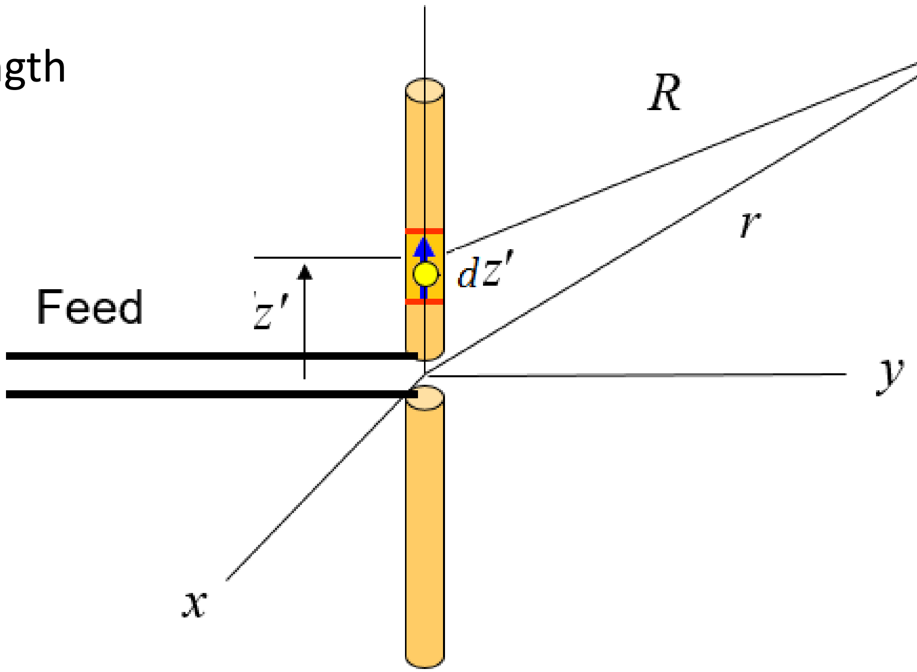


Half-wave



Wire Antenna (cont.)

Finite length



Far-field observation point (x, y, z)

$$at \quad r \geq \frac{2l^2}{\lambda}$$

For phase approximation is

$$R = r - z'\cos\theta$$

For distance approximation is

$$R = r$$

$$dE_\theta \simeq j\eta \frac{k\, I_e(z') e^{-jk(r - z'\cos\theta)}}{4\pi r} \sin\theta \, dz'$$

$$\simeq j\eta \frac{k\, I_e(z') e^{-jkr}}{4\pi r} \sin\theta \, e^{jkz'\cos\theta} dz' \quad (4\text{-}58)$$

Ideal Sinusoidal Current Distribution

$$\underline{I}_e = \begin{cases} \hat{a}_z I_0 \sin\left[k\left(\frac{l}{2} - z'\right)\right] & 0 \le z' \le l/2 \\ \hat{a}_z I_0 \sin\left[k\left(\frac{l}{2} + z'\right)\right] & -l/2 \le z' \le 0 \end{cases} \tag{4-56}$$

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$$E_\theta = j\eta \frac{ke^{-jkr}}{4\pi r}\sin\theta\left\{\int_{-l/2}^{0} I_o \sin\left[k\left(\frac{l}{2}+z'\right)\right]e^{+jkz'\cos\theta}dz' + \int_{0}^{+l/2} I_o \sin\left[k\left(\frac{l}{2}-z'\right)\right]e^{-jkz'\cos\theta}dz'\right\} \quad \text{(4-60)}$$

$$\int e^{\alpha x}\sin[\beta x+\gamma]dx = \frac{e^{\alpha x}}{\alpha^2+\beta^2}[\alpha\sin(\beta x+\gamma)-\beta\cos(\beta x+\gamma)] \quad \text{(4-61)}$$

$$\alpha = \pm jk\cos\theta, \qquad \beta = \pm k, \qquad \gamma = \frac{kl}{2} \qquad \text{(4-61a,b,c)}$$

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$$E_\theta = j\eta \frac{I_o e^{-jkr}}{2\pi r}\left[\frac{\cos\left(\frac{k\ell}{2}\cos\theta\right)-\cos\left(\frac{k\ell}{2}\right)}{\sin\theta}\right] \quad \text{(4-62a)}$$

$$E_\theta \cong C\left[\underbrace{\frac{\cos\left(\frac{k\ell}{2}\cos\theta\right)-\cos\left(\frac{k\ell}{2}\right)}{\sin\theta}}_{\text{Field Pattern}}\right]$$

$$H_\phi \cong \frac{E_\theta}{\eta}, \qquad C = j\eta \frac{I_o e^{-jkr}}{2\pi r}$$

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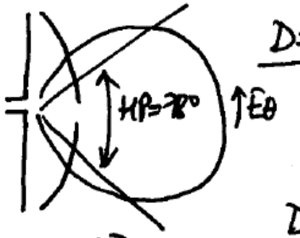
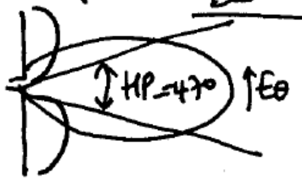
$$\Rightarrow \quad E_\theta = \frac{j\eta e^{-jkr}}{2\pi r} I_m \frac{\cos\left[\left(\frac{kL}{2}\right)\cos\theta\right] - \cos\left[\frac{kL}{2}\right]}{\sin\theta} \quad \& \quad H_\phi = \frac{E_\theta}{\eta}$$

for $L = \frac{\lambda}{2}$ (half-wave Dipole), $$E_\theta = \frac{j\eta e^{-jkr}}{2\pi r} I_m \frac{\cos\left[\left(\frac{\pi}{2}\right)\cos\theta\right]}{\sin\theta}$$



D=1.64

for $L = \lambda$ (Full-Wave Dipole), $$E_\theta = \frac{j\eta e^{-jkr}}{2\pi r} I_m \frac{\cos(\pi\cos\theta) + 1}{\sin\theta}$$



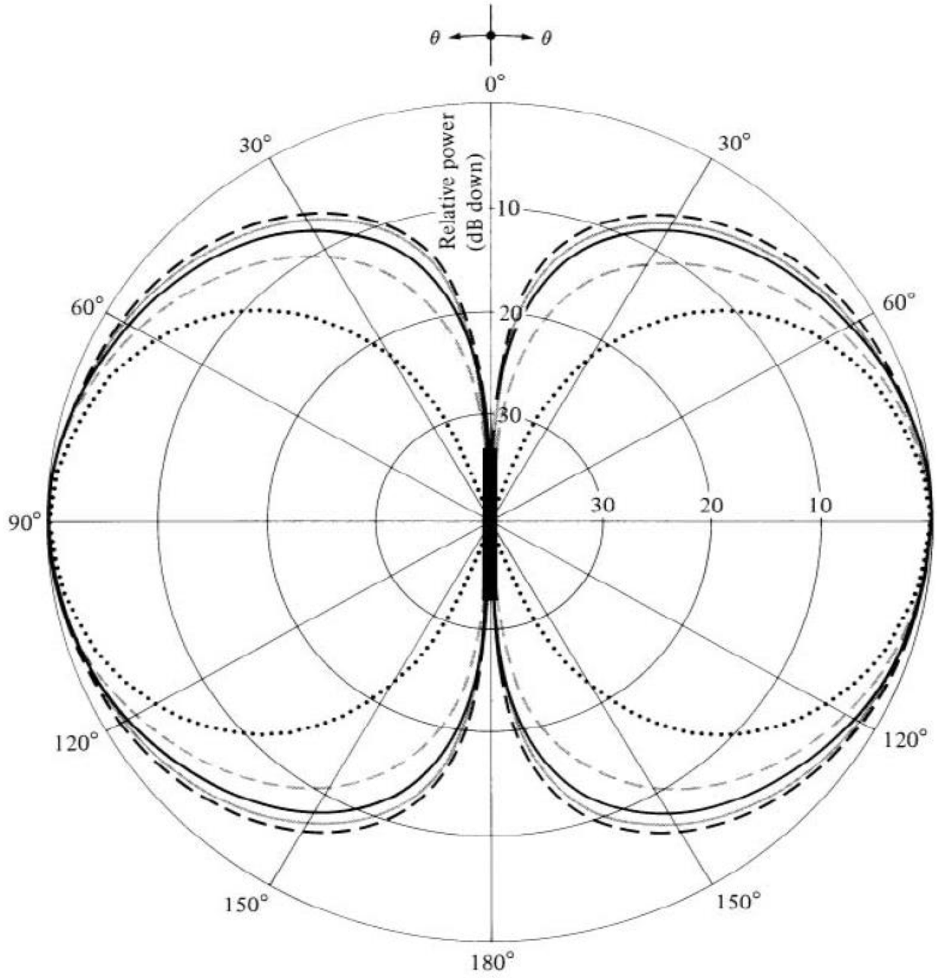
D=

for $L = \frac{3}{2}\lambda$ (1.5λ-long dipole), $$E_\theta = \frac{j\eta e^{-jkr}}{2\pi r} I_m \frac{\cos\left(\frac{3}{2}\pi\cos\theta\right)}{\sin\theta}$$

D=

Wire Antenna (cont.)

Results



$l = \lambda/50$	$l = \lambda/50$	3-dB beamwidth = 90°	
$l = \lambda/4$	$l = \lambda/4$	3-dB beamwidth = 87°	
$l = \lambda/2$	$l = \lambda/2$	3-dB beamwidth = 78°	
$l = 3\lambda/4$	$l = 3\lambda/4$	3-dB beamwidth = 64°	θh≈58
$l = \lambda$	$l = \lambda$	3-dB beamwidth = 47.8°	θh≈66

Wire Antenna (cont.)

radiation intensity

$$U = r^2 \, W_{av} = \eta \frac{|I_0|^2}{8\pi^2} \left[\frac{\cos\left(\frac{kl}{2}\cos\theta\right) - \cos\left(\frac{kl}{2}\right)}{\sin\theta} \right]^2 \quad (4\text{-}64)$$

Radiated power

$$P_{rad} = \int_0^{2\pi} \int_0^{\pi} W_{av} r^2 \sin\theta \, d\theta \, d\phi$$

$$= \eta \frac{|I_0|^2}{4\pi} \int_0^{\pi} \frac{\left[\cos\left(\frac{kl}{2}\cos\theta\right) - \cos\left(\frac{kl}{2}\right)\right]^2}{\sin\theta} d\theta \quad (4\text{-}67)$$

Given in the exam

$$P_{rad} = \eta \frac{|I_0|^2}{4\pi} \{C + \ln(kl) - C_i(kl) + \tfrac{1}{2}\sin(kl)[S_i(2kl) - 2S_i(kl)]$$
$$+ \tfrac{1}{2}\cos(kl)[C + \ln(kl/2) + C_i(2kl) - 2C_i(kl)]\} \quad (4\text{-}68)$$

where $C = 0.5772$ (Euler's constant) and $C_i(x)$ and $S_i(x)$ are the cosine and sine integrals (see Appendix III)

$$R_r = \frac{2P_{\mathrm{rad}}}{|I_0|^2} = \frac{\eta}{2\pi}\{C + \ln(kl) - C_i(kl)$$
$$+ \tfrac{1}{2}\sin(kl) \times [S_i(2kl) - 2S_i(kl)]$$
$$+ \tfrac{1}{2}\cos(kl) \times [C + \ln(kl/2) + C_i(2kl) - 2C_i(kl)]\}$$

For half wave length dipole: $kl=\pi$

Appendix III

x	$S_i(x)$	$C_i(x)$	$C_{in}(x)$
6.2	1.41871	−0.03587	2.43764
6.3	1.41817	−0.01989	2.43765
3.1	1.85166	0.08699	1.62163
3.2	1.85140	0.05526	1.68511

$R_r = 60*1.217 = 73.05\ \Omega$

$R_{rad} \approx 73\ [\Omega]$

Input resistance for finite dipole

For lossless antenna input resistance contain radiation resistance where $R_{loss}=0$
the radiation resistance is referred to the maximum current which for some lengths ($3\lambda/4$, λ) does not occur at the input terminals of the antenna

$$P_{in}=P_{rad}$$

$$\frac{|I_{in}|^2}{2}R_{in} = \frac{|I_o|^2}{2}R_r$$

where

R_{in} = radiation resistance at input (feed) terminals

I_{in} = current at input terminals

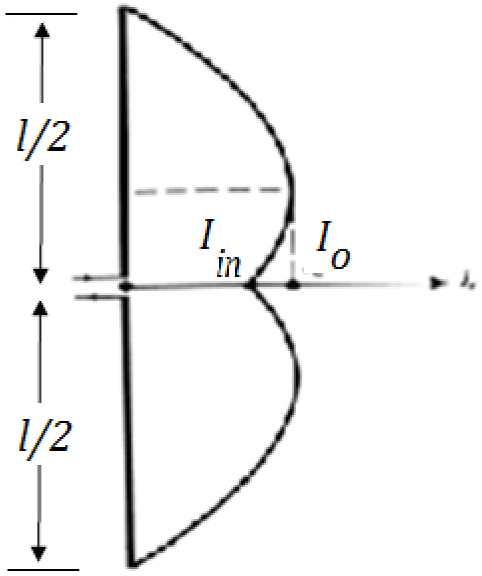
R_r = radiation resistance at current maximum

I_0 = current maximum

For dipole of length $l \leq \boldsymbol{\lambda}$

$$I_{in} = I_o sin\left(\frac{kl}{2}\right)$$

$$R_{in} = \frac{R_r}{sin^2\left(\frac{kl}{2}\right)}$$



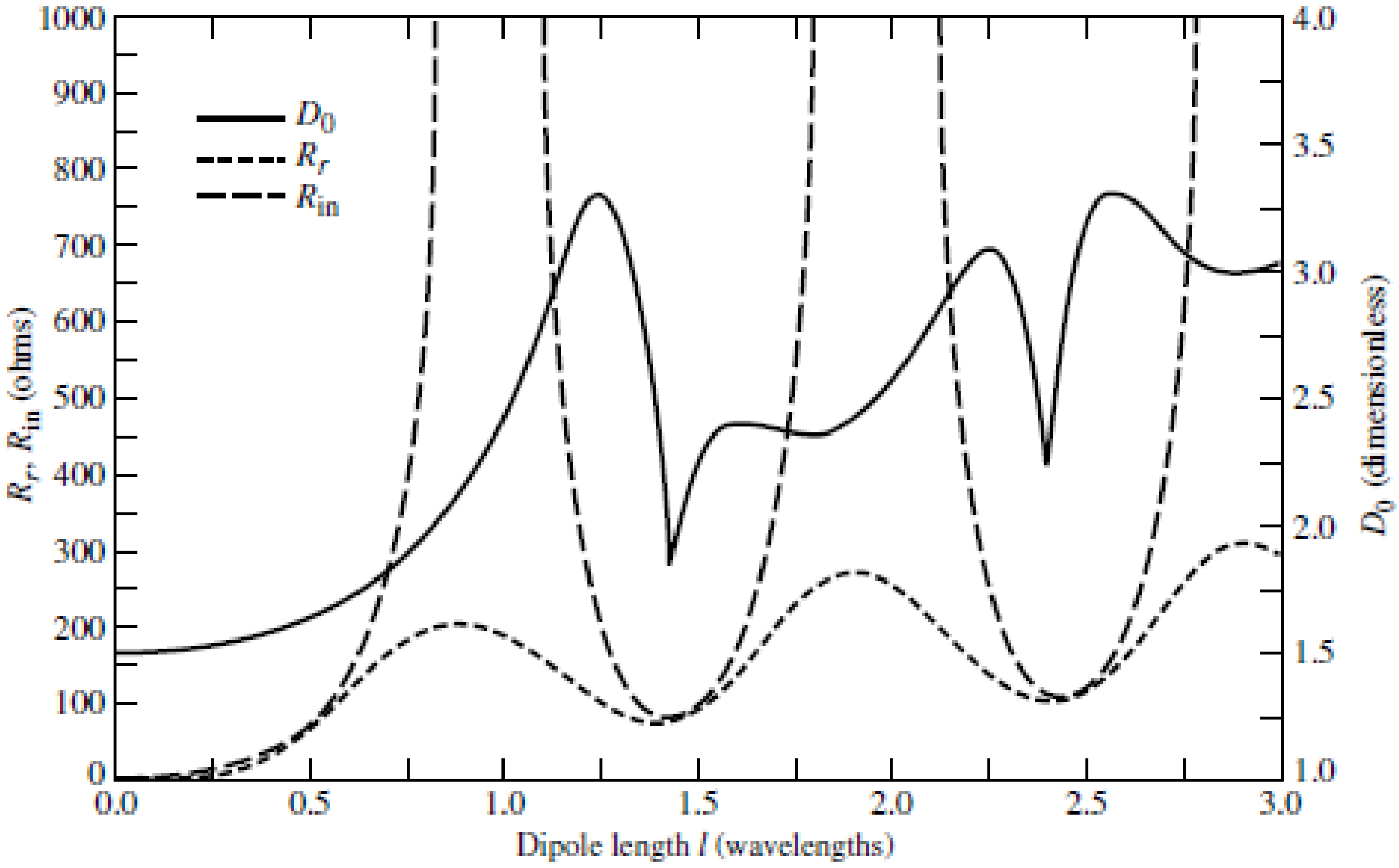


Figure 4.9 Radiation resistance, input resistance and directivity of a thin dipole with sinusoidal current distribution.

Directivity

the radiation pattern of a dipole becomes more directional as its length increases.
When the overall length is greater than one wavelength, the number of lobes increases and the antenna loses its directional properties.

$$F(\theta) = \left[\frac{\cos\left(\frac{kl}{2}\cos\theta\right) - \cos\left(\frac{kl}{2}\right)}{\sin\theta} \right]^2$$

$$D_0 = \frac{2F(\theta)|_{\max}}{Q}$$

Valid for all finite length dipole

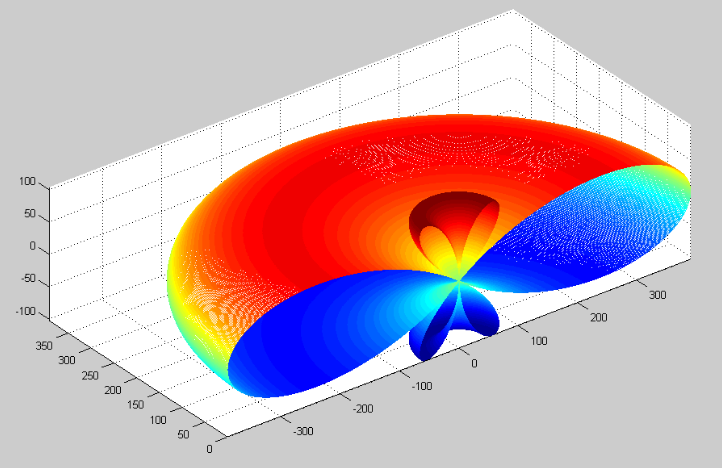
$$Q = \{C + \ln(kl) - C_i(kl) + \tfrac{1}{2}\sin(kl)[S_i(2kl) - 2S_i(kl)] + \tfrac{1}{2}\cos(kl)[C + \ln(kl/2) + C_i(2kl) - 2C_i(kl)]\} \qquad (4\text{-}75a)$$

For half wavelength dipole

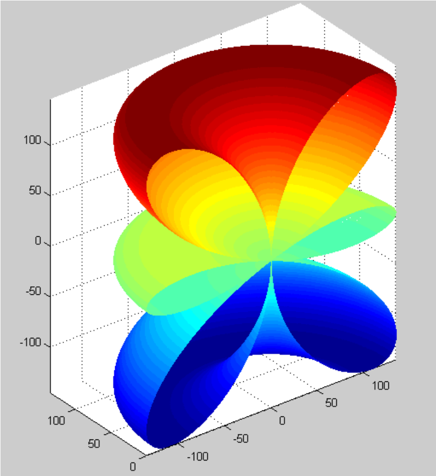
$$D_0 = \frac{2}{1.217} = 1.64 \quad , \quad A_{em} = \frac{\lambda^2}{4\pi} D_0$$

Normalized amplitude pattern for dipoles of lengths >λ

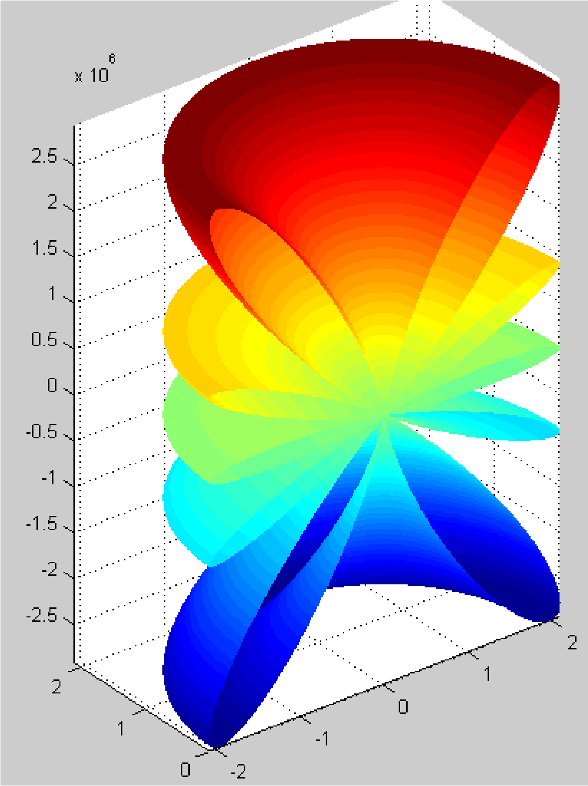
L=1.25λ



L=1.5λ



L=5λ



dipoleCH4.m

Monopoles and Dipoles

- Monopoles and dipoles are widely used antennas in wireless communications systems.
- Monopoles are particularly popular for portable units and on automobiles and other vehicles.
- In practice, wide use is made of the *quarter-wavelength monopole*.

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Monopoles and Dipoles

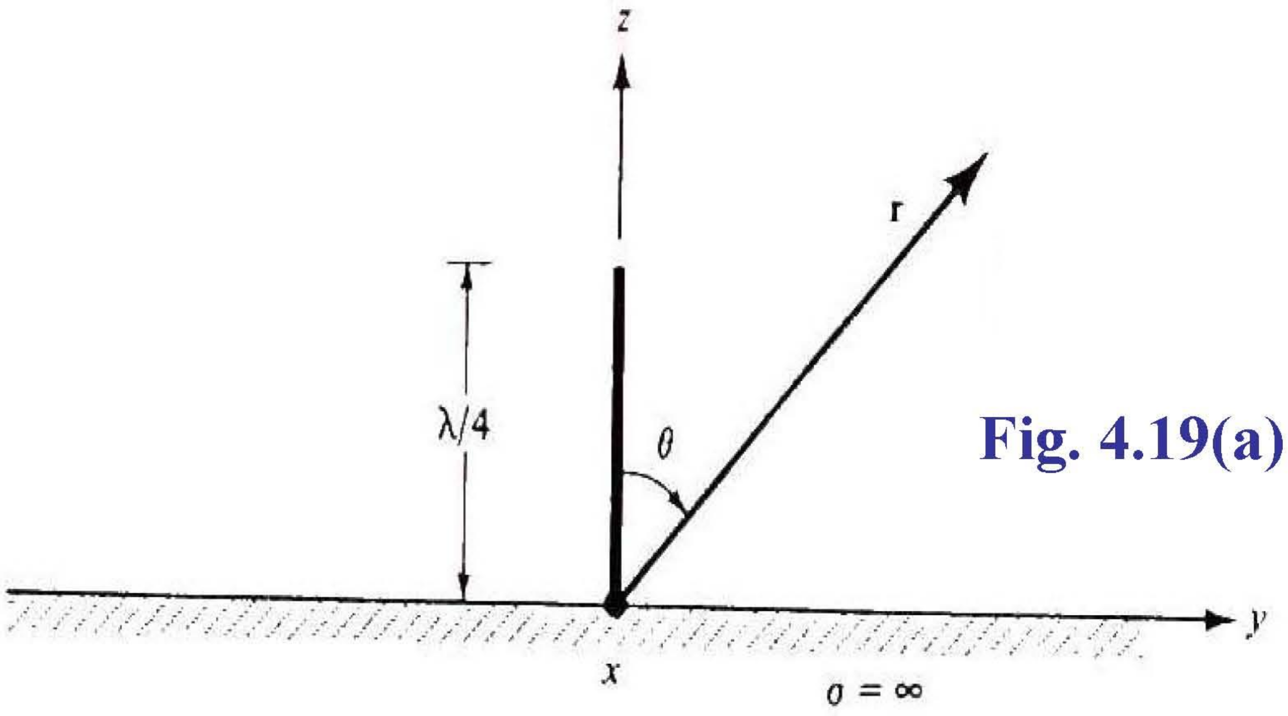

Fig. 4.19(a)

(a) λ/4 monopole on infinite electric conductor

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Monopoles and Dipoles

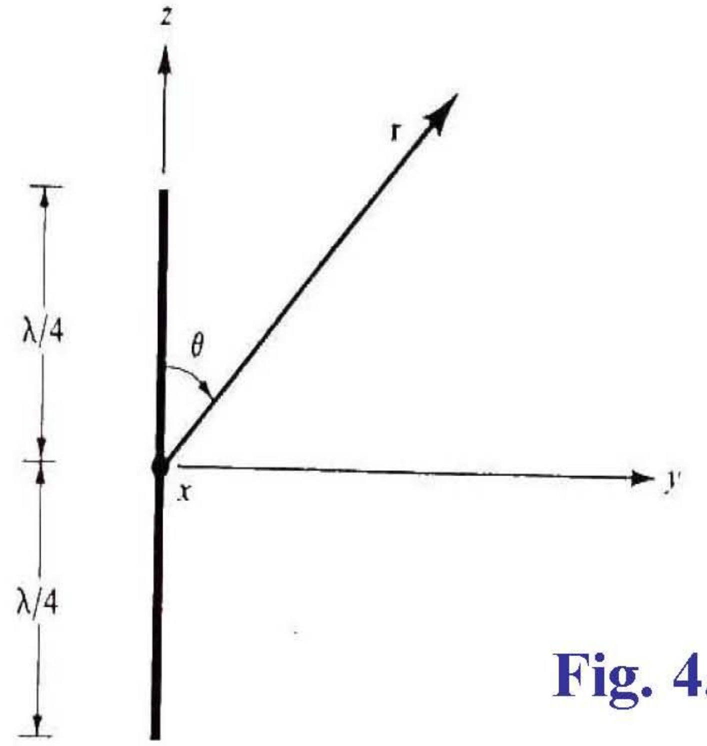

Fig. 4.19(b)

(b) Equivalent of λ/4 monopole on infinite electric conductor

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Monopoles and Dipoles

$$Z_{in}\ \text{(monopole)} = \frac{1}{2} Z_{in}\ \text{(dipole)}$$

$$D_0\ \text{(monopole)} = 2D_0\ \text{(dipole)} \qquad (4\text{-}106)$$

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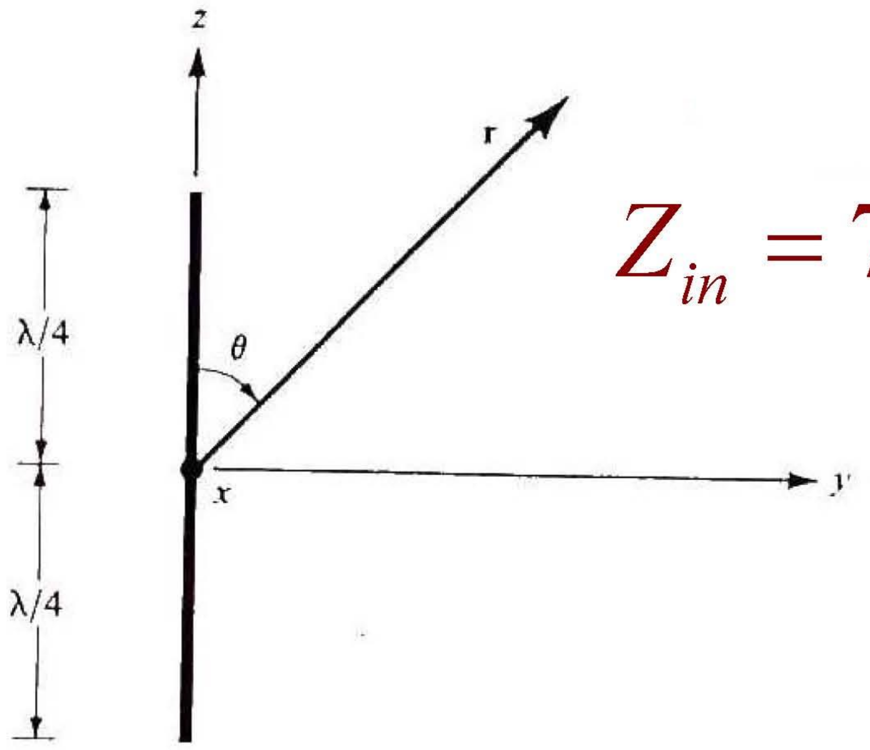
For $\lambda/2$ Dipole



$$Z_{in} = 73 + j42.5$$

Fig. 4.19(b)

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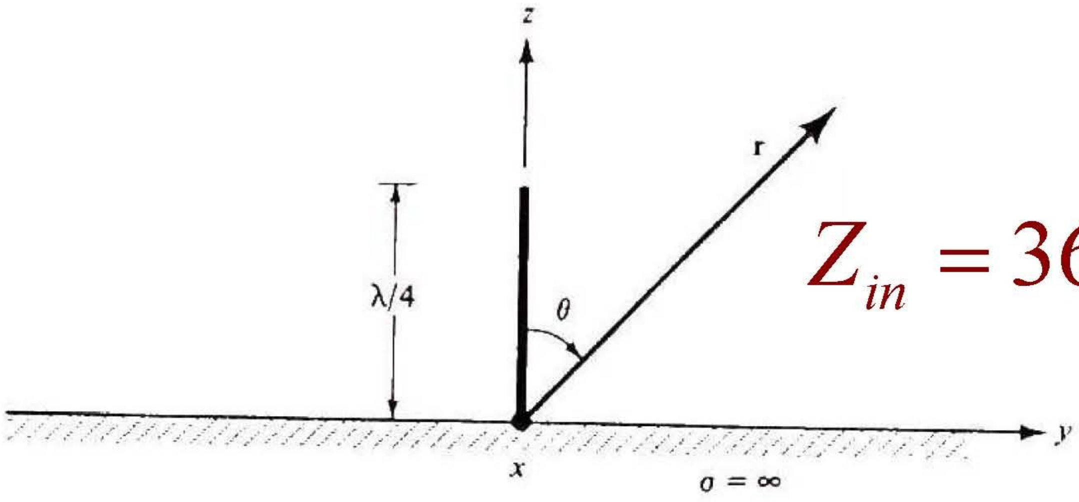
λ/4 Monopole on Infinite Electric Conductor



$$Z_{in} = 36.5 + j21.25$$

Fig. 4.19(a)

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